令和8年度　創造工学専攻　学力検査

数 学　　解答用紙　　受検番号

問1

(1) $\boxed{5}$

$y \neq 0 \Rightarrow$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\downarrow \int dx$$

$$\log|y| = \tan^{-1}x + C$$

$$|y| = e^{\tan^{-1}x + C} = e^C e^{\tan^{-1}x}$$

$$y = \underbrace{\pm e^C}_{= C} e^{\tan^{-1}x}$$

$$y = C e^{\tan^{-1}x}.$$ //

$y=0$ は $C=0$ とした解

(2) $\boxed{5}$

$$(D^2+4D+5)y = 8\sin x$$

$$(D^2+4D+5)y_0 = 0$$

$$\| $$

$$((D+2)^2+1)y_0$$

$$\updownarrow \ D = -2 \pm i$$

$$y_0 = e^{-2x}[C_0\cos x + C_1\sin x]$$ //

$$(D^2+4D+5)y_1 = 8\sin x$$

$$\downarrow \quad \uparrow \mathrm{Im}$$

$$(D^2+4D+5)\tilde{y}_1 = 8e^{ix}$$

$$\updownarrow$$

$$\tilde{y}_1 = 8\frac{1}{D^2+4D+5}e^{ix}$$

$$= 8e^{ix}\frac{1}{4}\frac{1}{1+i}$$

$$= 2\frac{1}{1+i}\frac{1-i}{1-i}(\cos x + i\sin x)$$

$$= \cos x + \sin x + i(-\cos x + \sin x)$$

$$y_1 = \mathrm{Im}\,\tilde{y}_1 = -\cos x + \sin x$$ //

$$y = y_0 + y_1$$

$$= e^{-2x}[C_0\cos x + C_1\sin x] - \cos x + \sin x$$ //

問 2

(1) $\boxed{4}$

$$f=\frac{1}{3}x^3+xy^2-x+1$$

$$\begin{cases} f_x = x^2+y^2-1=0 \\ f_y = 2xy = 0 \end{cases}$$

$x=0$ or $y=0$

$x=0 \Rightarrow y=\pm 1$

$y=0 \Rightarrow x=\pm 1$

$(x,y)=(\pm1,0),(0,\pm1)$ //

(2) $\boxed{3}$

$$H(x,y)=\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}(x,y)$$

$$=\begin{vmatrix} 2x & 2y \\ 2y & 2x \end{vmatrix}$$

$$=4(x^2-y^2)$$

$f_{xx}(x,y)=2x$

$H(0,\pm1)=-4<0$

$H(\pm1,0)=4>0$

$f_{xx}(\pm1,0)=\pm2 \begin{matrix} >0 \\ <0 \end{matrix} \begin{matrix} \cup \\ \cap \end{matrix}$

$f(\pm1,0)=\begin{cases} \frac{1}{3} & \text{極小値} \\ \frac{5}{3} & \text{極大値} \end{cases}$ //

(3) $\boxed{3}$

$$\frac{1}{3}x^3+xy^2-x+1=0$$

$$\downarrow \frac{d}{dx}$$

$$x^2+y^2+2xy\frac{dy}{dx}-1=0$$

$$2xy\frac{dy}{dx}=1-x^2-y^2$$

$$\Updownarrow$$

$$\frac{dy}{dx}=\frac{1-x^2-y^2}{2xy} \quad \begin{pmatrix} x\neq0, \\ y\neq0 \end{pmatrix}$$

問 3

(1) $\boxed{4}$

$$x^3+x^2+x+1=(x+1)(x^2+1)$$

$$\frac{2}{(x+1)(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}$$

$\times(x+1)$ $x\to -1$ $A=1.$

$\times(x^2+1)$ $x\to i$ $\frac{2}{1+i}=Bi+C$

$$C+iB=\frac{2}{1+i}\frac{1-i}{1-i}=1+i(-1)$$

$B=-1,\ C=1$

$$\frac{2}{x^3+x^2+x+1}=\frac{1}{x+1}+\frac{-x+1}{x^2+1}$$ //

(2) $\boxed{4}$

$$\int\frac{2}{x^3+x^2+x+1}dx$$

$$=\int\left(\frac{1}{x+1}-\frac{x}{x^2+1}+\frac{1}{x^2+1}\right)dx$$

$$=\log|x+1|-\frac{1}{2}\log(x^2+1)+\tan^{-1}x+C$$

$$=\log\frac{|x+1|}{\sqrt{x^2+1}}+\tan^{-1}x+C$$ //

(3) $\boxed{2}$

$$\int_0^\infty\frac{2}{x^3+x^2+x+1}dx$$

$$=0+\frac{\pi}{2}-\{0+0\}$$

$$=\frac{\pi}{2}$$ //

問4

(1) $\boxed{5}$

$$\iint_D y^2\,dx\,dy$$

$$D=\{0\leq y\leq -x+2,\ 0\leq x\leq 2\}$$

$$=\int_0^2 \underbrace{\int_0^{-x+2} y^2\,dy}_{\left[\frac{1}{3}y^3\right]_0^{-x+2}=\frac{1}{3}(-x+2)^3}\,dx$$

$$=\int_0^2 \frac{1}{3}(-x+2)^3\,dx$$

$$=\frac{1}{3}\left[-\frac{1}{4}(-x+2)^4\right]_0^2$$

$$=\frac{1}{3}\left[0-\left(-\frac{1}{4}2^4\right)\right]$$

$$=\frac{4}{3}\ _{//}$$

(2) $\boxed{5}$

$$\iint_D y^2\,dx\,dy \qquad \begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$$

$$D=\{0<r<2,\ 0<\theta<\tfrac{\pi}{2}\}$$

$$=\int_0^{\frac{\pi}{2}}\int_0^2 (r\sin\theta)^2\,r\,dr\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\underbrace{\int_0^2 r^3\,dr}_{\frac{1}{4}2^4=2^2}\,\sin^2\theta\,d\theta$$

$$=2^2\,\frac{1}{2}\,\frac{\pi}{2}$$

$$=\pi\ _{//}$$

問 5 $\boxed{10}$

$$
\begin{array}{c}
\left(\begin{array}{cccc|c}
1 & -1 & -1 & 1 & 1 \\
1 & 1 & 3 & -2 & -5 \\
-1 & 2 & 1 & -1 & 6 \\
1 & 1 & 1 & 1 & 17
\end{array}\right)
\end{array}
\begin{array}{l}
 \\ \times -1 \\ \times 1 \\ \times -1
\end{array}
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & -1 & -1 & 1 & 1 \\
0 & 2 & 4 & -3 & -6 \\
0 & 1 & 0 & 0 & 7 \\
0 & 2 & 2 & 0 & 16
\end{array}\right)
\begin{array}{l}
\times 1 \\ \times -2 \\ \\ \times -2
\end{array}
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & -1 & 1 & 8 \\
0 & 1 & 0 & 0 & 7 \\
0 & 0 & 4 & -3 & -20 \\
0 & 0 & 2 & 0 & 2
\end{array}\right)
\begin{array}{l}
 \\ \\ \\ \times \frac{1}{2}
\end{array}
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & -1 & 1 & 8 \\
0 & 1 & 0 & 0 & 7 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 4 & -3 & -20
\end{array}\right)
\begin{array}{l}
\times 1 \\ \\ \\ \times -4
\end{array}
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 1 & 9 \\
0 & 1 & 0 & 0 & 7 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & \cancel{-3}\ 1 & \cancel{-24}\ 8
\end{array}\right)
\begin{array}{l}
\times -1 \\ \\ \\ \times -\frac{1}{3}
\end{array}
$$

$$
\rightarrow
\left(\begin{array}{cccc|c}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 7 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 8
\end{array}\right)
\qquad
\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 1 \\ 8 \end{pmatrix} /\!/
$$

問6

$$A=\begin{pmatrix}2&3&3\\3&2&3\\3&3&2\end{pmatrix}$$

(1) $\boxed{5}$

$$0=|\lambda I-A|$$

$$=\begin{vmatrix}\lambda-2&-3&-3\\-3&\lambda-2&-3\\-3&-3&\lambda-2\end{vmatrix}\ \times 1$$

$$=(\lambda-8)\begin{vmatrix}1&1&1\\-3&\lambda-2&-3\\-3&-3&\lambda-2\end{vmatrix}\ \times 3$$

$$=(\lambda-8)\begin{vmatrix}1&1&1\\0&\lambda+1&0\\0&0&\lambda+1\end{vmatrix}$$

$$=(\lambda-8)(\lambda+1)^2$$

$\Updownarrow$

$\lambda=-1$ (重), 8 //

(2) $\boxed{5}$

$(\lambda I-A)x=0$ をとく.

$\lambda=-1$ (重) $\Rightarrow$

$$\left(\begin{array}{ccc|c}-3&-3&-3&0\\-3&-3&-3&0\\-3&-3&-3&0\end{array}\right)$$

$$x+y+z=0$$

$$x=t\begin{pmatrix}1\\0\\-1\end{pmatrix}+s\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

$(t,s\in\mathbb{R})$ //

$\lambda=8\Rightarrow$

$$\left(\begin{array}{ccc|c}6&-3&-3&0\\-3&6&-3&0\\-3&-3&6&0\end{array}\right)\ \times 1,\ \times -\tfrac{1}{3},\ \times -\tfrac{1}{3}$$

$$\rightarrow\left(\begin{array}{ccc|c}0&0&0&0\\1&-2&1&0\\1&1&-2&0\end{array}\right)\ \times -1$$

$$\rightarrow\left(\begin{array}{ccc|c}0&0&0&0\\1&-2&1&0\\0&\cancel{3}&\cancel{-3}&0\end{array}\right)\ \times 2$$

$1\quad -1\quad 0$

$$\rightarrow\left(\begin{array}{ccc|c}0&0&0&0\\1&0&-1&0\\0&1&-1&0\end{array}\right)$$

$$x=t\begin{pmatrix}1\\1\\1\end{pmatrix}\quad(t\in\mathbb{R})$$

$\lambda=-1$ (重) $\Rightarrow$ $x=\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}0\\1\\-1\end{pmatrix}$ 　もしくは $\begin{pmatrix}-1\\1\\0\end{pmatrix},\begin{pmatrix}-1\\0\\1\end{pmatrix}$

$\lambda=8\Rightarrow x=\begin{pmatrix}1\\1\\1\end{pmatrix}$ //

基底が正しいなら正答

問7

(1) $\boxed{5}$

$$\begin{vmatrix} a_1 & a_2 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$\times -2$　$\times -3$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & -3 & -6 \\ 7 & -6 & -12 \end{vmatrix}$$

$\times -2$

$= 0$ より

$\{a_1, a_2, a_3\}$ は一次従属 //

(2) $\boxed{5}$

$$\begin{vmatrix} a_1 & a_2 & a_3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -4 & 3 \\ 3 & -5 & 6 \end{vmatrix}$$

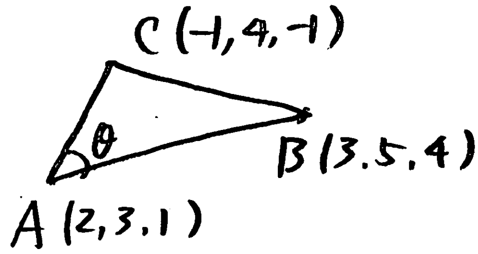
$\times 1$　$\times -2$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 2 & -2 & -1 \\ 3 & -2 & 0 \end{vmatrix}$$

$= -2 \neq 0$ より

$\{a_1, a_2, a_3\}$ は一次独立 //

問 8



$O(0,0,0)$

(1) $\boxed{3}$ $\overrightarrow{OA}=\begin{pmatrix}2\\3\\1\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}3\\5\\4\end{pmatrix}$.

$$\overrightarrow{OA}\times\overrightarrow{OB}=\begin{pmatrix}12-5\\3-8\\10-9\end{pmatrix}=\begin{pmatrix}7\\-5\\1\end{pmatrix}$$

$$\begin{pmatrix}7\\-5\\1\end{pmatrix}\cdot\begin{pmatrix}x-0\\y-0\\z-0\end{pmatrix}=0.$$

$$7x-5y+z=0 \ //$$

(2) $\boxed{3}$ $\theta=\angle BAC$.

$\overrightarrow{AB}=\begin{pmatrix}1\\2\\3\end{pmatrix}$. $\overrightarrow{AC}=\begin{pmatrix}-3\\1\\-2\end{pmatrix}$.

$$|\overrightarrow{AB}|=|\overrightarrow{AC}|=\sqrt{14}$$

$$\overrightarrow{AB}\cdot\overrightarrow{AC}=-3+2-6=-7.$$

$$\cos\theta=\frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|}=-\frac{1}{2}$$

$$\Updownarrow$$

$$\theta=\frac{2}{3}\pi \ //$$

(3) $\boxed{3}$

$$S=\frac{1}{2}|\overrightarrow{AB}||\overrightarrow{AC}|\cdot\sin\theta$$

$$=\frac{1}{2}\cdot\sqrt{14}^2\cdot\frac{\sqrt{3}}{2}=\frac{\cancel{14}^{7}}{2\cdot\cancel{2}}\sqrt{3}$$

$$S=\frac{7}{2}\sqrt{3} \ //$$

(4) $\boxed{1}$

$$G\left(\frac{2+3-1}{3},\ \frac{3+5+4}{3},\ \frac{1+4-1}{3}\right)$$

より

$$G\left(\frac{4}{3},\ 4,\ \frac{4}{3}\right) \ //$$

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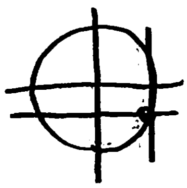
問 9

(1) $\boxed{5}$

$$y = \sqrt{3}\sin x - \cos x$$

$$= 2\left[\sin x \times \underbrace{\frac{\sqrt{3}}{2}}_{\cos\theta} + \cos x \underbrace{\left(-\frac{1}{2}\right)}_{\sin\theta}\right]$$



$\theta = -\frac{\pi}{6}$

より

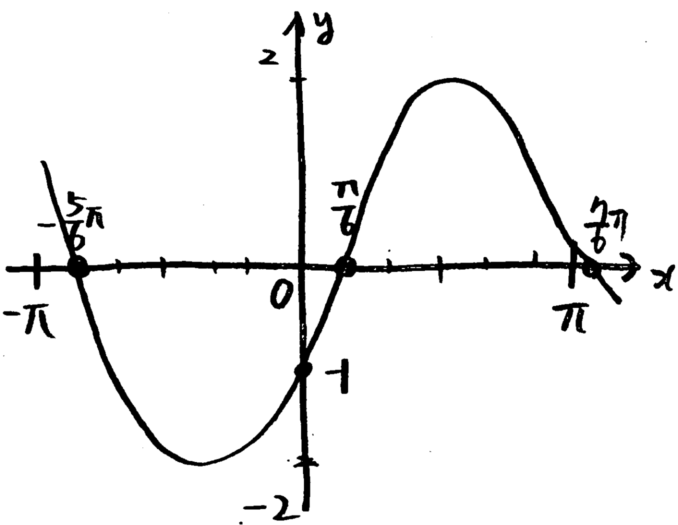
$$y = 2\sin\left(x - \frac{\pi}{6}\right) \;/\!/$$

(2) $\boxed{5}$

$y = 0 \Rightarrow x - \frac{\pi}{6} = -\pi,\ 0,\ \pi$

$x = -\frac{5}{6}\pi,\ \frac{\pi}{6}\ \left(\frac{7}{6}\pi\right)$

$x = 0 \Rightarrow y = 2\sin\left(-\frac{\pi}{6}\right) = -1$



問 10

(1) $\boxed{3}$

$$\frac{13}{52} = \frac{1}{4} //$$

(2) $\boxed{3}$

1st

$$\frac{13}{52} \times \frac{12}{51} = \frac{\cancel{13}}{4 \times \cancel{13}} \times \frac{\cancel{3} \times 4}{\cancel{3} \times 17} = \frac{1}{17} //$$

(3) $\boxed{2}$

1st ♡　2nd ♡

$$\frac{13}{52} \times \frac{12}{51} = \frac{1}{17} \leftarrow (2)$$

1st ♡以外 2nd ♡

$$\frac{39}{52} \times \frac{13}{51} = \frac{3}{4} \cdot \frac{13}{3 \cdot 17} = \frac{13}{4 \cdot 17}$$

より

$$\frac{1}{17} + \frac{13}{4 \cdot 17} = \frac{17}{4 \cdot 17} = \frac{1}{4} //$$

(4) $\boxed{2}$

A　1st ♡

B　2nd ♡

$$P(A) = \frac{1}{4},\quad P(B) = \frac{1}{4},\quad P(A \cap B) = \frac{1}{17}$$

$$P_B(A) = \frac{P(B \cap A)}{P(B)} = \frac{\frac{1}{17}}{\frac{1}{4}} = \frac{4}{17} //$$